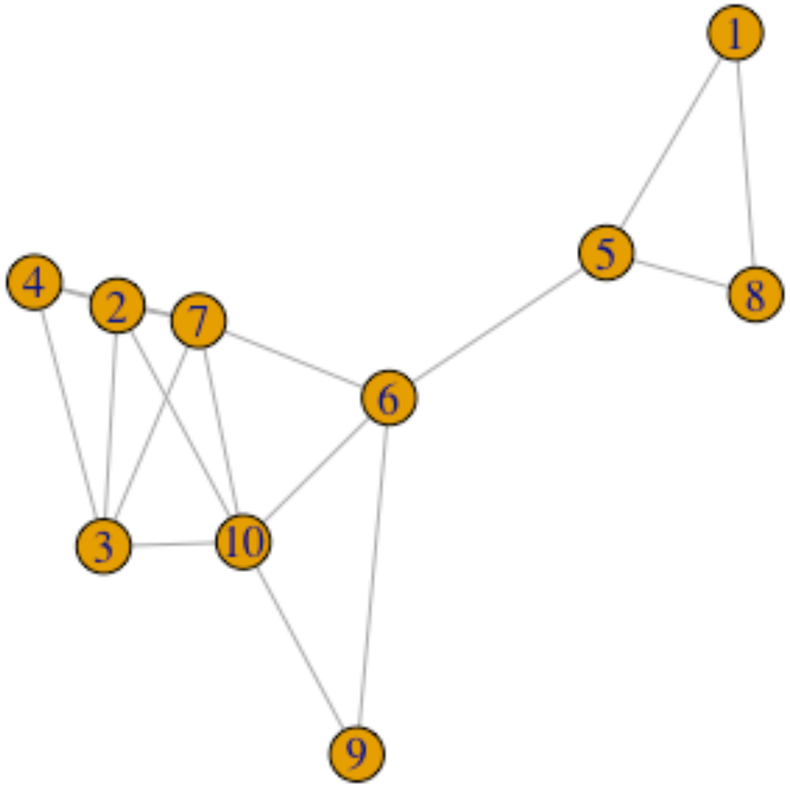
Introduction to graphons

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CSIRO

NeurIPS Education Track 2026

A graph is ...

- Undirected
- Unweighted
- No loops
- No node or edge attributes

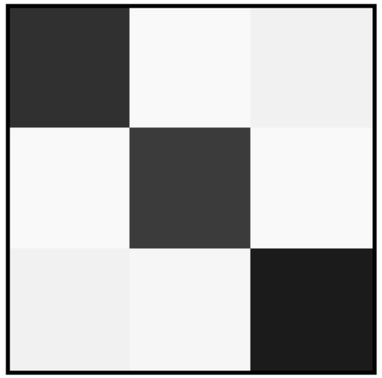


Graphons as graph generators

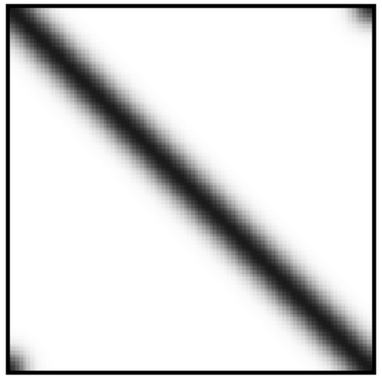


What are graphons?

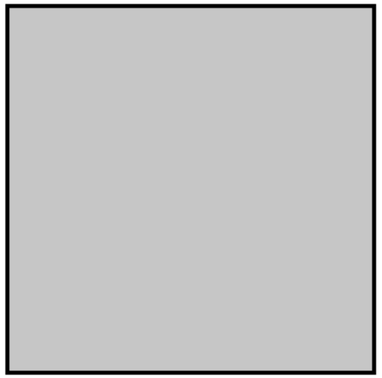
- A symmetric, measurable function $W : [0,1]^2 \to [0,1]$



Stochastic Block Model graphon



Ring graphon

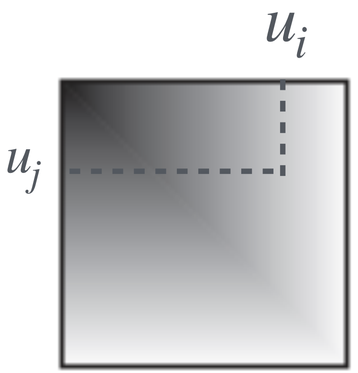


Erdos-Renyi graphon

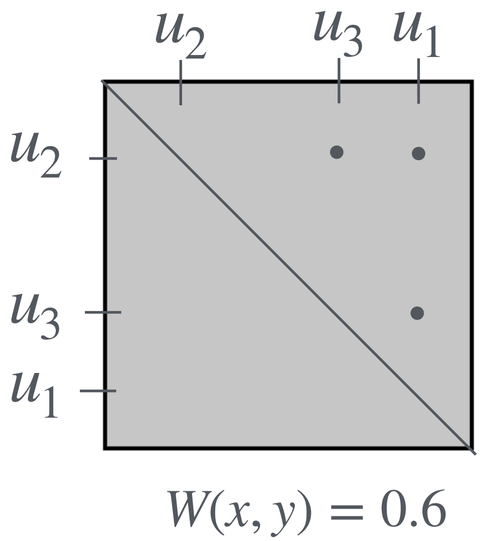
- Compact domain, bounded function, no need to be continuous

Sampling graphs from a graphon

- Say your graph has n nodes labeled $\{1,2,\ldots,n\}$
- Uniformly sample n points from the interval $[0,1]$ - $\{u_1, u_2, \ldots, u_n\}$
- For every (u_i, u_j) the value $W(u_i, u_j)$ is the edge probability
- Edge $e_{ij} \sim$ Bernoulli$(W(u_i, u_j))$
- Toss a coin with probability $W(u_i, u_j)$ and connect the nodes i and j if you get heads
- Each edge is connected independently - we'll get back to this later on!

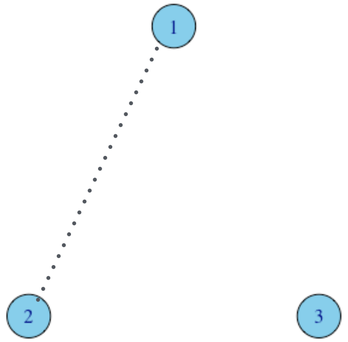


Example



$W(x, y) = 0.6$

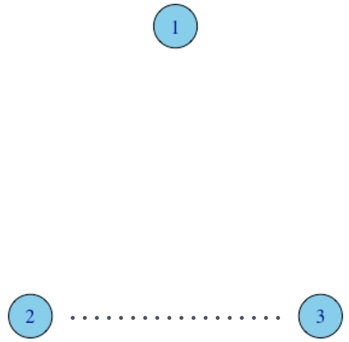
Consider edge e_{21}



Bernoulli$(W(u_2, u_1)) = 0$

=> No edge

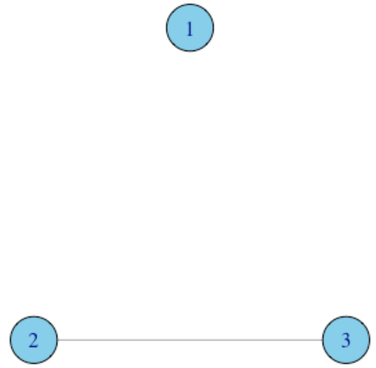
Consider edge e_{23}



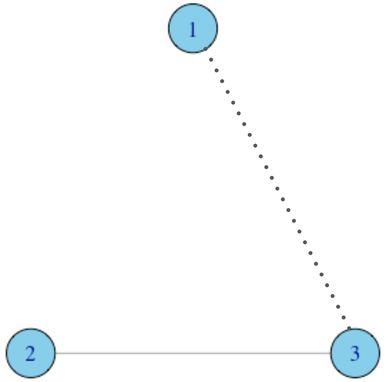
Bernoulli$(W(u_2, u_3)) = 1$

=> Add edge

Resulting graph



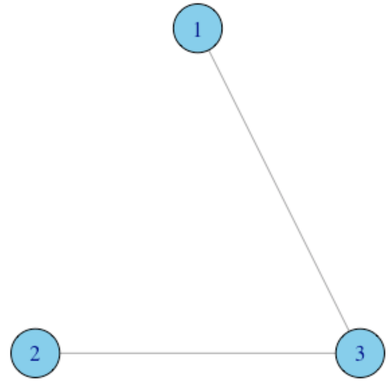
Consider edge e_{31}



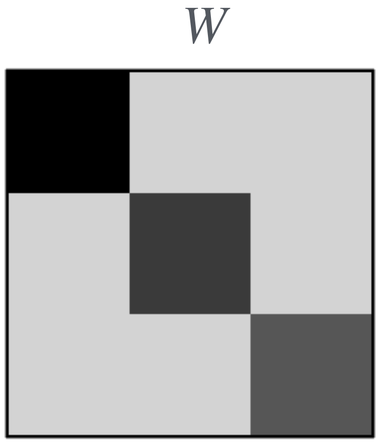
Bernoulli$(W(u_3, u_1)) = 1$

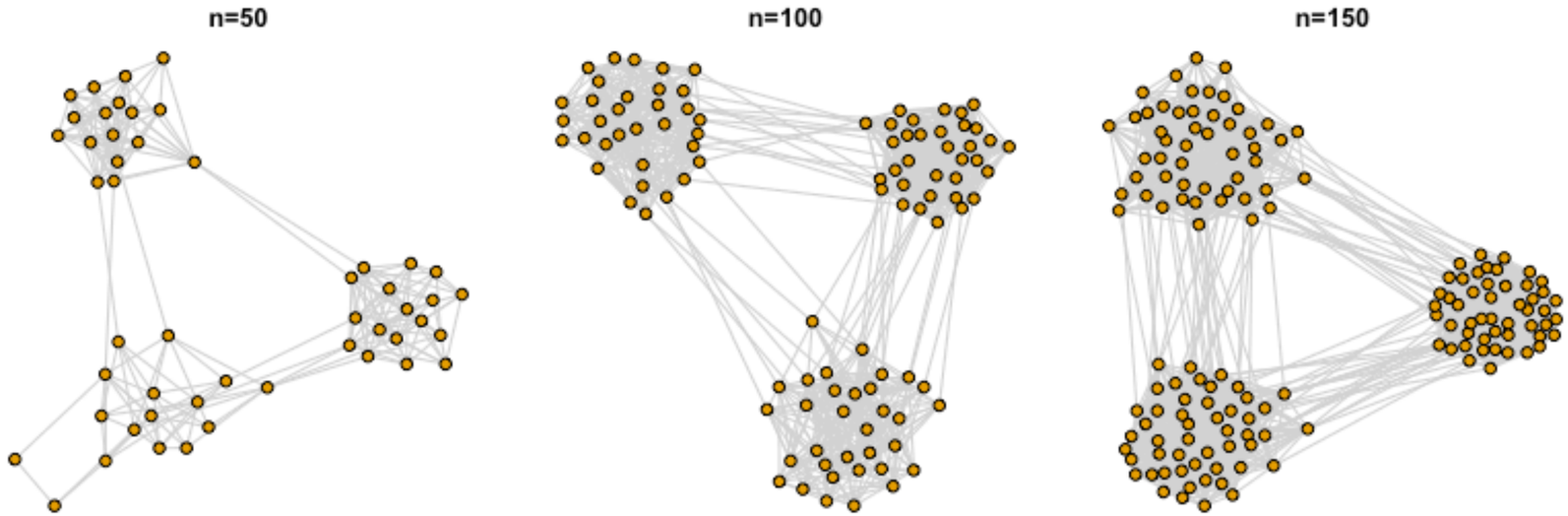
=> Add edge

Resulting graph



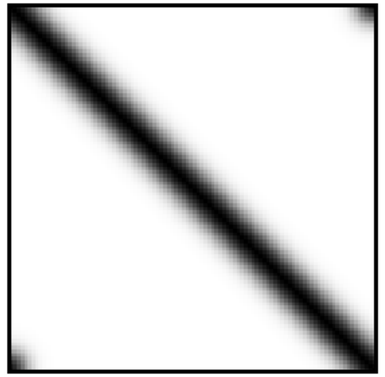
Stochastic block model graphon

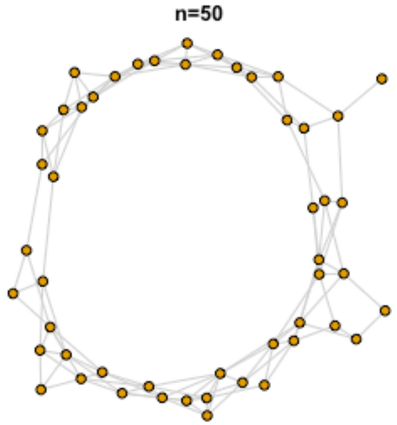




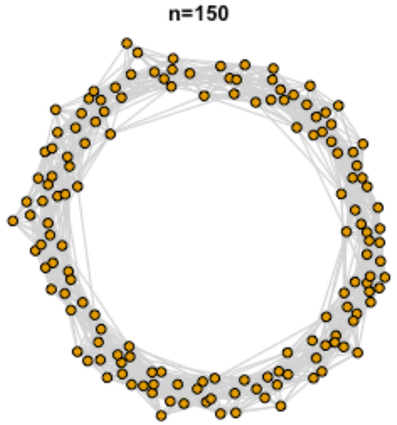
Ring graphon

W

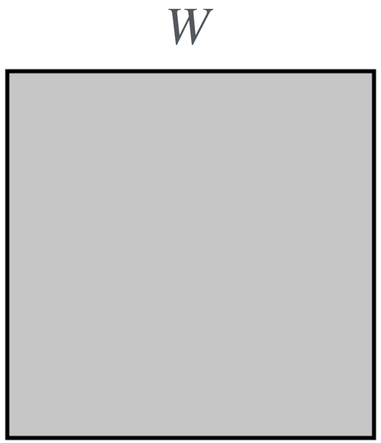


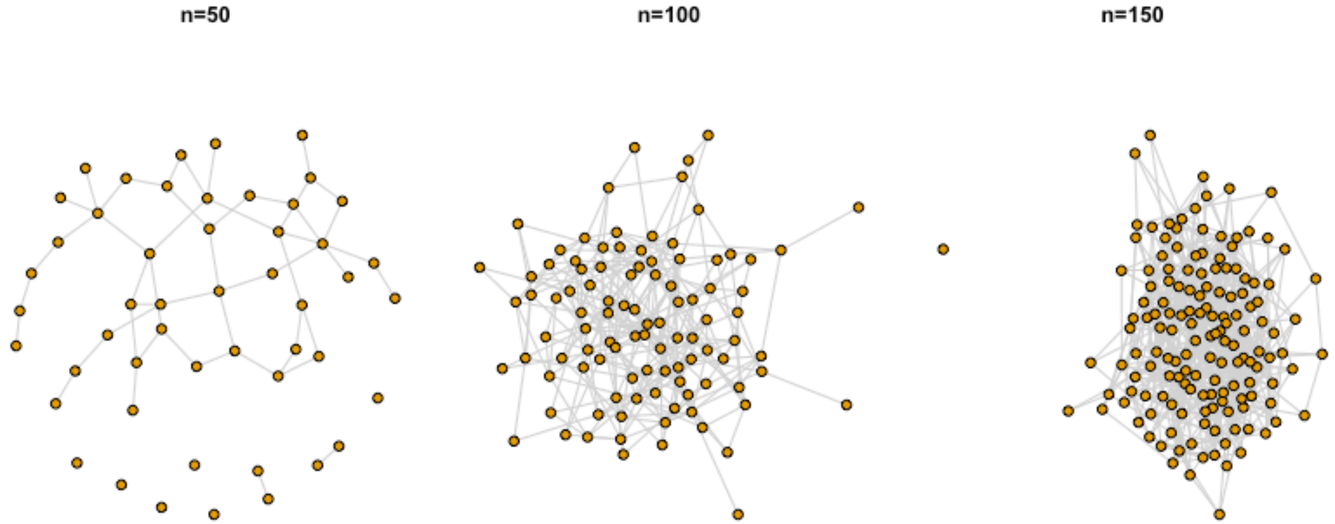






Constant graphon



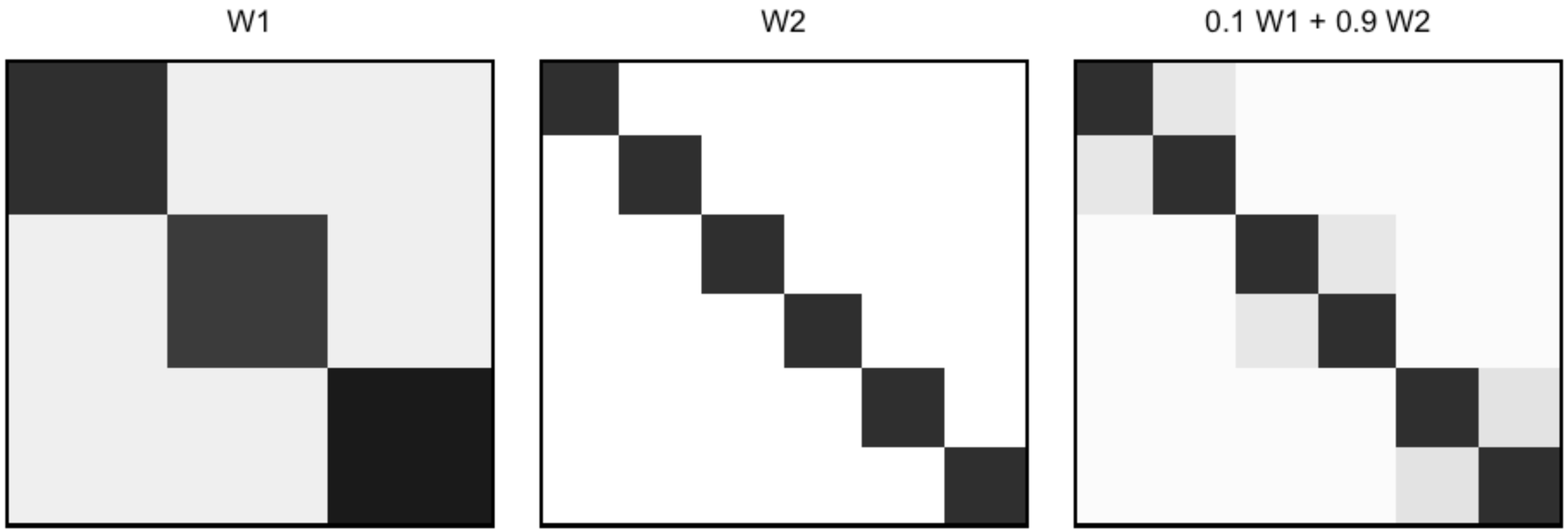


Combining graphons



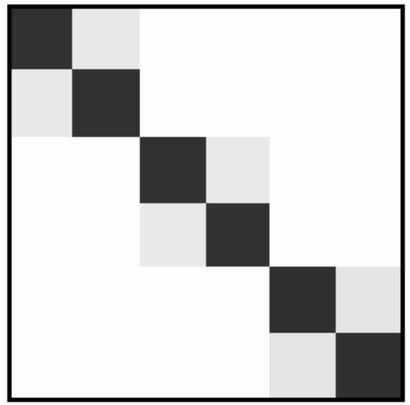
Convex combination of graphons

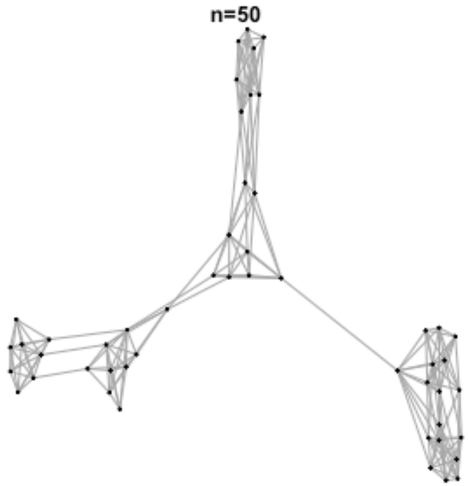
- Let W_1 and W_2 be graphons
- Consider $W = \alpha W_1 + (1 - \alpha) W_2$ for $\alpha \in [0,1]$

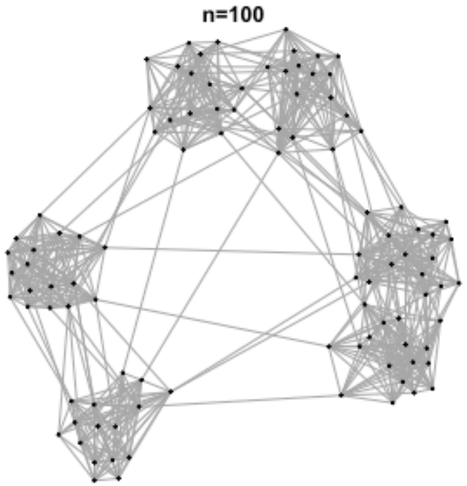


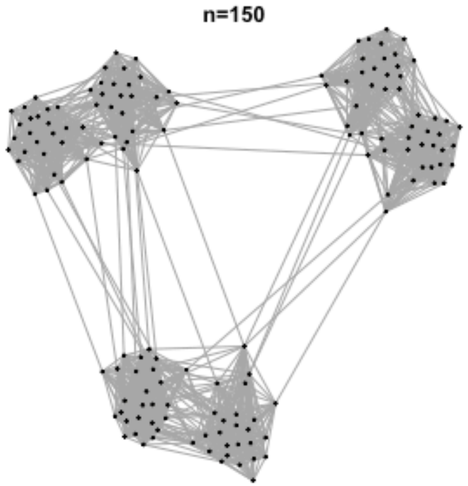
Convex combination

W



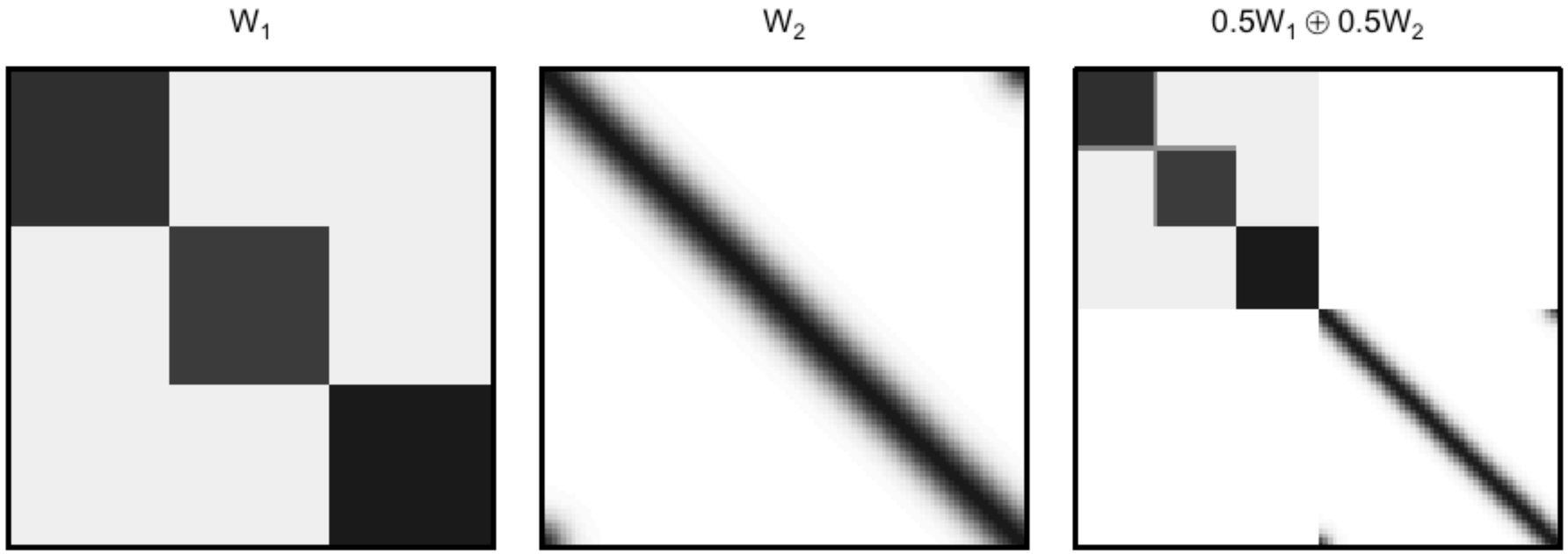






Disjoint union of graphs

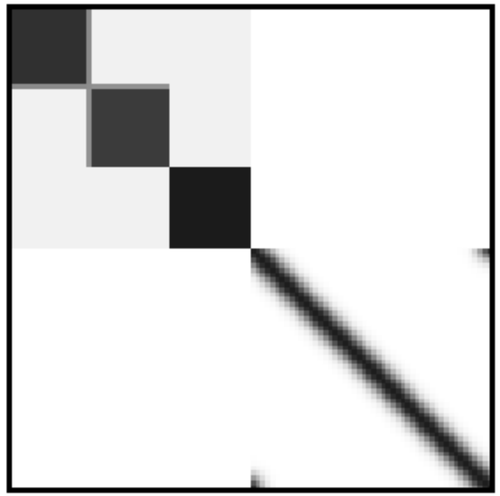
- Let W_1 and W_2 be graphons
- Let $W = \alpha W_1 \oplus (1-\alpha) W_2$ for $\alpha \in [0,1]$ visualized by placing a scaled graphon W_1 on $[0,\alpha]^2$ and a scaled graphon W_2 on $[\alpha,1]^2$



Janson, S. (2008). Connectedness in graph limits. *arXiv preprint arXiv:0802.3795*.

Disjoint union - example 1

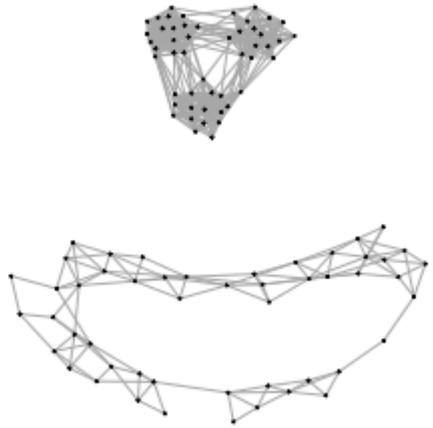
W



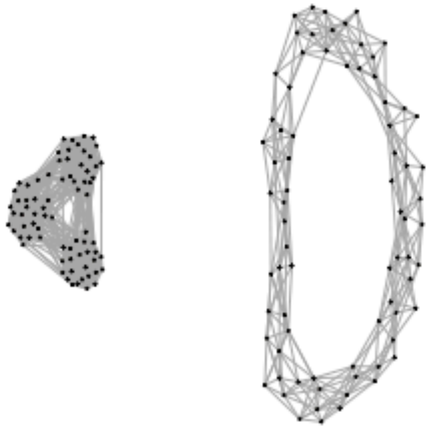
n=70



n=100

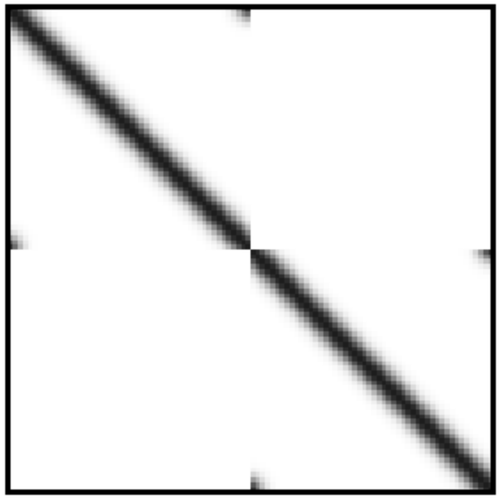


n=150



Disjoint union - example 2

W



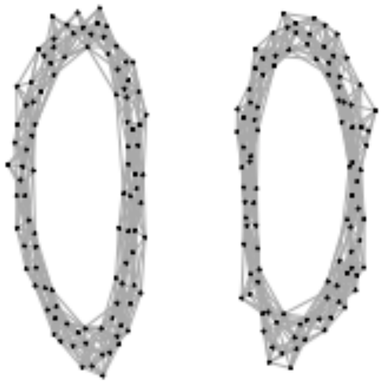
n=100



n=150

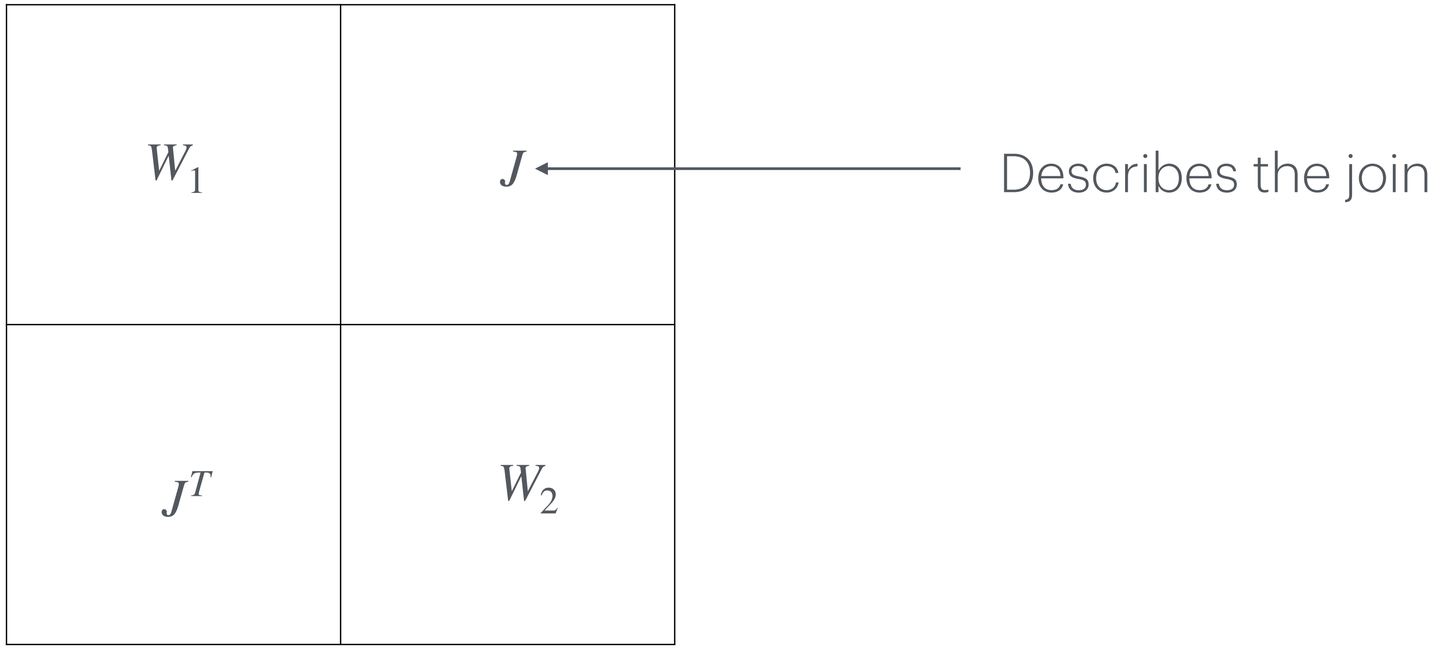


n=200



Joining disjoint unions

- Let W_1 and W_2 be graphons, and $J : [0,1]^2 \to [0,1]$, but J does not need to be symmetric
- Scale W_1, W_2 and J as necessary.



Joined graphs

Describes the join

