Some Approaches for Sparse Graphs

Sevvandi Kandanaarachchi, Cheng Soon Ong
CSIRO

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Kallenberg Exchangeable Graphs

- Introduced by Caron and Fox (2017) and later used by many others
- Nodes $\theta_i \in \mathbb{R}^+$
- $W : \mathbb{R}_+^2 \to [0,1]$
  defined on the positive quadrant

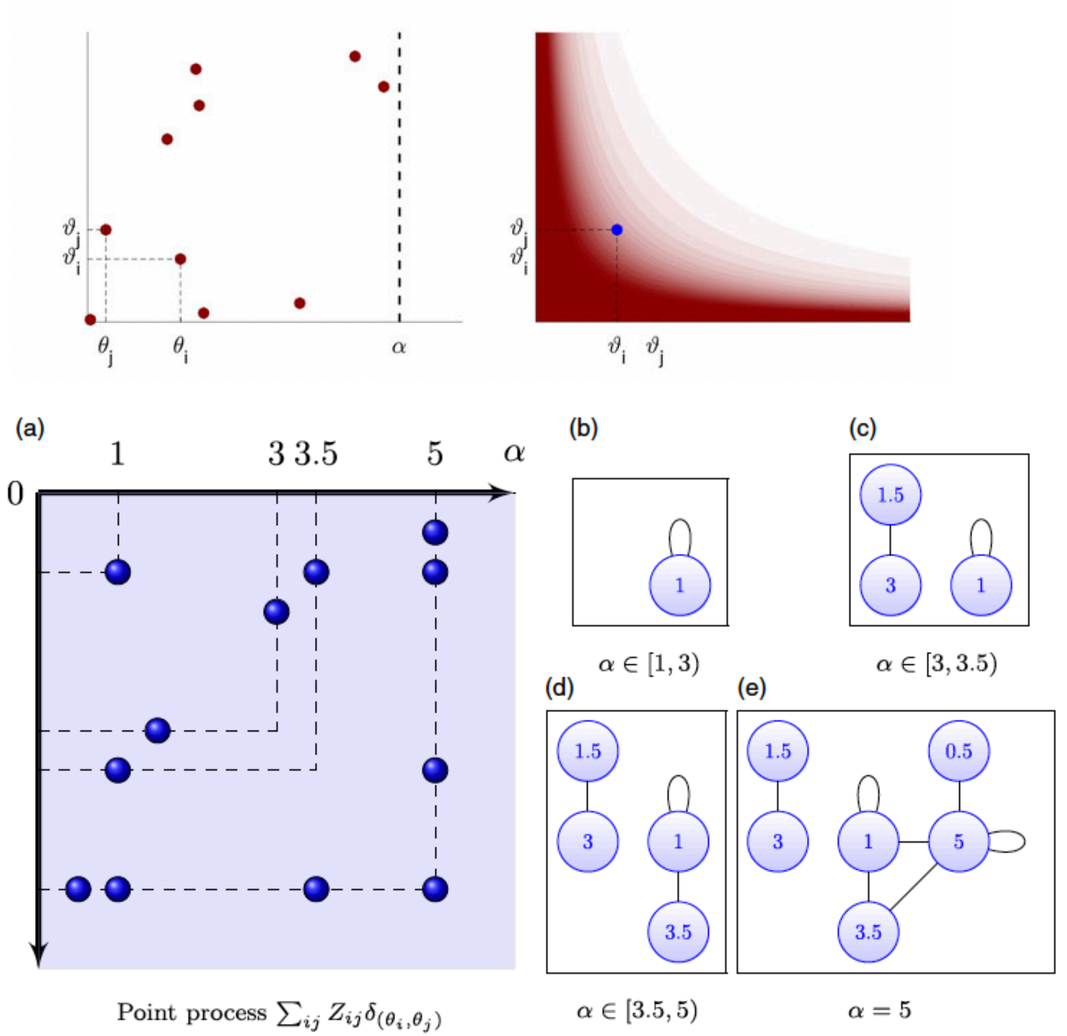


FIGURE 3. Illustration of the connection between the point process on the plane and the graphex process. (a) Point process $\sum_{ij} Z_{ij}\delta_{(\theta_i,\theta_j)}$ on the plane. (b–e) Associated graphs $\mathcal{G}_\alpha$ for (b) $\alpha \in [1, 3)$, (c) $\alpha \in [3, 3.5)$, (d) $\alpha \in [3.5, 5)$, and (e) $\alpha = 5$. Note that the graph is empty for $\alpha < 1$.

Caron, F., & Fox, E. B. (2017). Sparse graphs using exchangeable random measures. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, *79*(5), 1295-1366.

Caron, F., Panero, F., & Rousseau, J. (2023). On sparsity, power-law, and clustering properties of graphex processes. *Advances in Applied Probability*, *55*(4), 1211-1253.

Rescaled and stretched graphons

- Many methods proposed by Borgs & Chayes research group
- Rescaled $\widetilde{W} = \frac{W}{\|W\|_1}$
- Stretched graphon defined on $\mathbb{R}^2_+$.
  $\widetilde{W}(x,y) = W(\|W\|_1^{0.5}x, \|W\|_1^{0.5}y)$

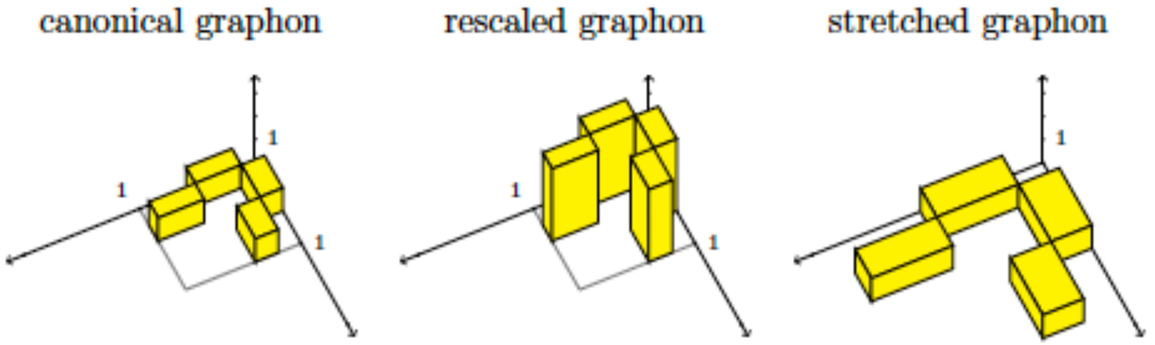


Figure 3: The figure shows three graphons associated with the same simple graph G on five vertices. In the classical theory of graphons all simple sparse graphs converge to the zero graphon. We may prevent this by renormalizing the graphons, either by rescaling the height of the graphon (middle) or by stretching the domain on which it is defined (right). The first approach was chosen by Bollobás and Riordan (2009) and Borgs, Chayes, Cohn, and Zhao (2014a,b), and the second approach is chosen in this paper. In our forthcoming paper (Borgs, Chayes, Cohn, and Holden, 2017) we choose a combined approach, where the renormalization depends on the observed graph.

Borgs, C., Chayes, J. T., Cohn, H., & Holden, N. (2018). Sparse exchangeable graphs and their limits via graphon processes. *Journal of Machine Learning Research*, *18*(210), 1-71.

Edge exchangeability

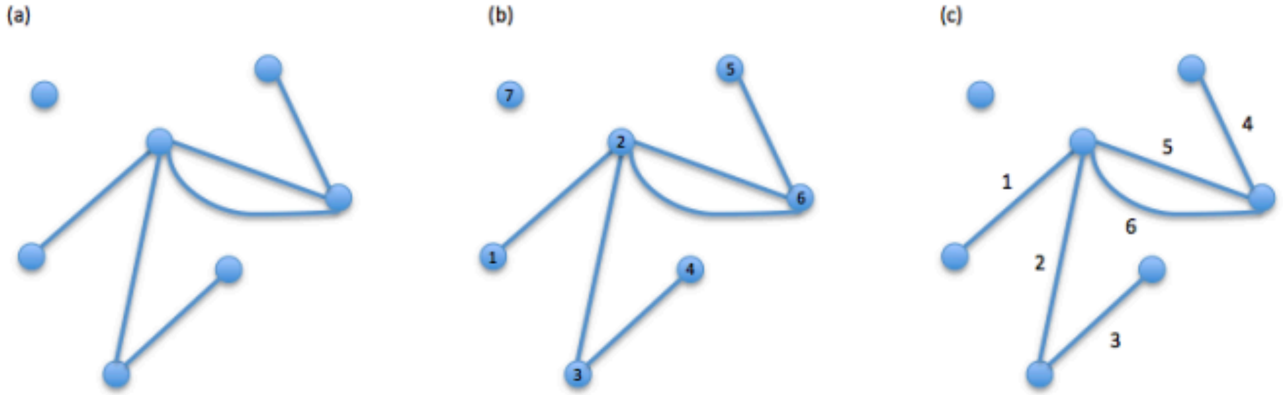


FIGURE 1. (a) A network structure derived from some process of interactions. Neither vertices nor edges come equipped with labels. (b) Network obtained by labeling vertices of network in Panel (a). (c) Network obtained by labeling edges of network in Panel (a).

Crane, H., & Dempsey, W. (2015). A framework for statistical network modeling. *arXiv preprint arXiv:1509.08185*.

Crane, H., & Dempsey, W. (2016). Edge exchangeable models for network data. *arXiv preprint arXiv:1603.04571*.

Cai, D., Campbell, T., & Broderick, T. (2016). Edge-exchangeable graphs and sparsity. *Advances in Neural Information Processing Systems*, *29*.

Janson, S. (2018). On edge exchangeable random graphs. *Journal of statistical physics*, *173*(3), 448-484.

Graphexes

- Originally introduced by Veitch & Roy
- A triple (I, S, W) where $I \in \mathbb{R}$ models isolated edges, $S : \mathbb{R}_+ \to \mathbb{R}_+$ integrable, models infinite stars and $W : \mathbb{R}_+^2 \to [0,1]$ is the standard graphon. But they consider $I = 0$ and $S = 0$
- Borgs and Chayes group - high degree hubs give the skeleton with low degree vertices forming the body

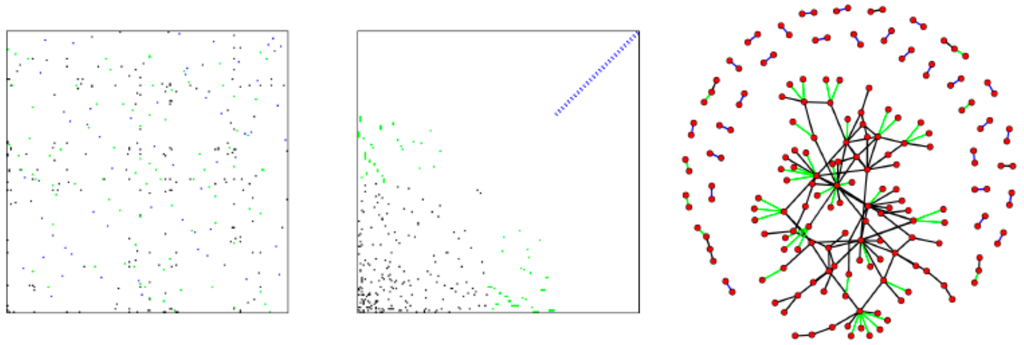


FIGURE 3. Realization of unlabeled graphex process generated by $\mathcal{W} = (I, S, W)$ at size $s = 15$ (right panel), and associated dilated empirical graphon (left and center panels). The generating graphex is $W = (x+1)^{-2}(y+1)^{-2}$, $S = 1/2\exp(-(x+1))$, and $I = 0.1$. The observation size is $s = 15$. The dilated empirical graphex is

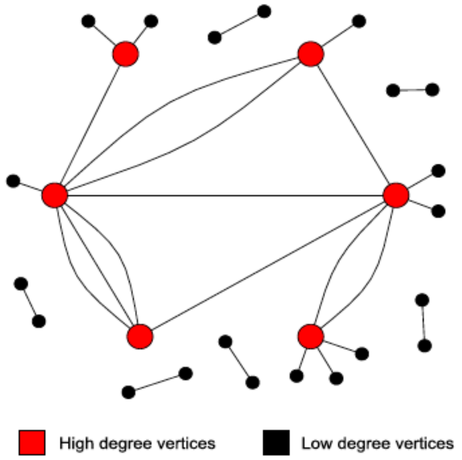


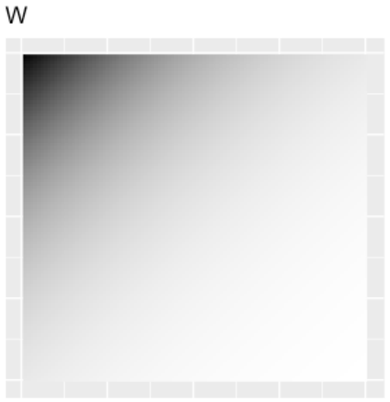
FIG 1. *Typical structure of* $\mathrm{Smpl}(\mathrm{CM}_n(\boldsymbol{d}), t/\sqrt{\ell_n})$.

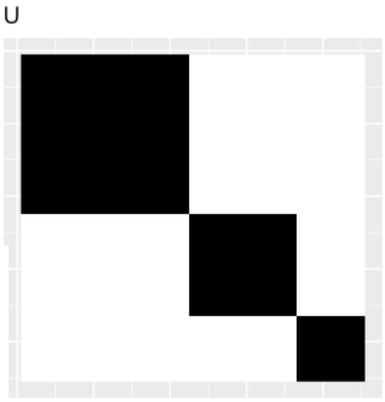
Veitch, V., & Roy, D. M. (2015). The class of random graphs arising from exchangeable random measures. *arXiv preprint arXiv:1512.03099*.

Veitch, V., & Roy, D. M. (2019). Sampling and estimation for (sparse) exchangeable graphs.

Borgs, C., Chayes, J. T., Dhara, S., & Sen, S. (2021). Limits of sparse configuration models and beyond. *The Annals of Probability*, *49*(6), 2830-2873.

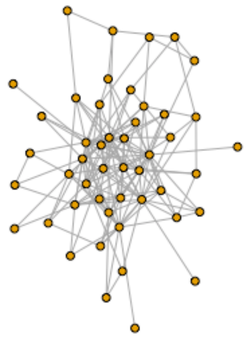
Graphon mixtures

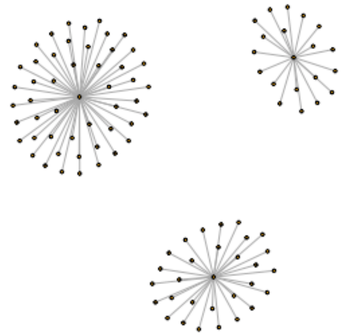




Sample dense graph

G_{n_d}





Mixture graph after joined



Sample sparse graph

$G_{n_s} = L^{-1}(H_{m_s})$

First sample disjoint cliques

Then compute the inverse line graph

- Parameters: dense nodes n_d, sparse edges m_s, the number of edges when joining m_{new}

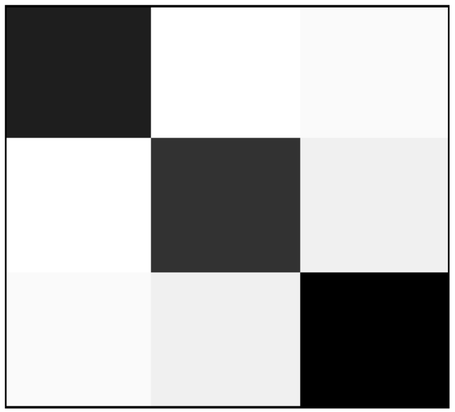
Kandanaarachchi, S., & Ong, C. S. (2026). Graphon Mixtures. *AISTATS, 2026*

Graphettes

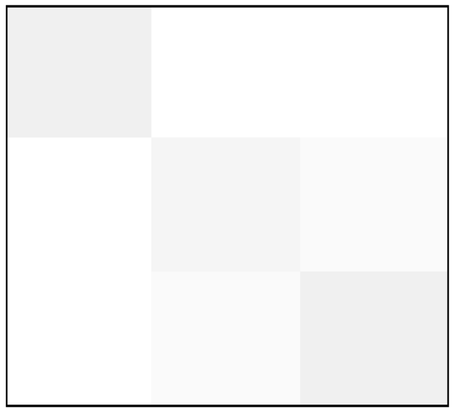
- Graphette $\mathscr{W} = (W, \rho_n, f)$
- W is a graphon on $[0,1]^2$
- $\{\rho_n\}_n = (\rho_1, \ldots, \rho_i, \ldots)$, a sequence, can converge to zero
- f is a graph edit function - eg. ring addition, star addition, cycle deletion

Wijesinghe, A., Kandanaarachchi, S., Steinberg, D. M., & Ong, C. S. (2026). Flowette: Flow Matching with Graphette Priors for Graph Generation. *arXiv preprint arXiv:2602.23566*.

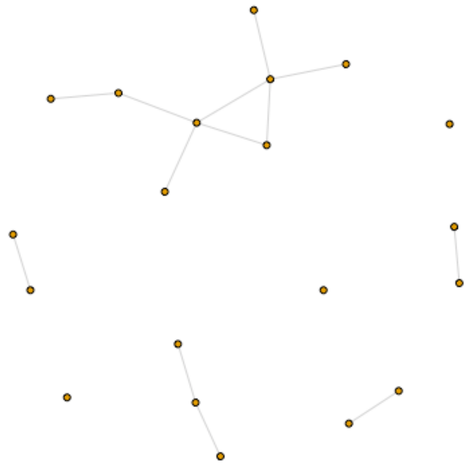
Example $\mathscr{W} = (W, \rho_n, f)$ with $\rho_n = c/n$ and $f = R$



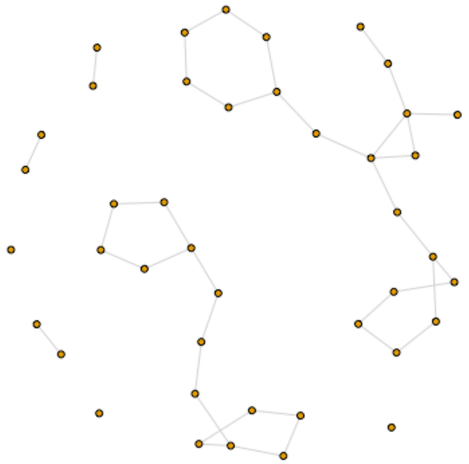
W



$W_n = \rho_n W$



$\tilde{G}_n$



$G_n = R(\tilde{G}_n)$

- R denotes ring addition - Bernoulli r.v.
- First $\tilde{G}_n \sim W_n = \rho_n W$ and then $G_n = R(\tilde{G}_n)$

Wijesinghe, A., Kandanaarachchi, S., Steinberg, D. M., & Ong, C. S. (2026). Flowette: Flow Matching with Graphette Priors for Graph Generation. *arXiv preprint arXiv:2602.23566*.