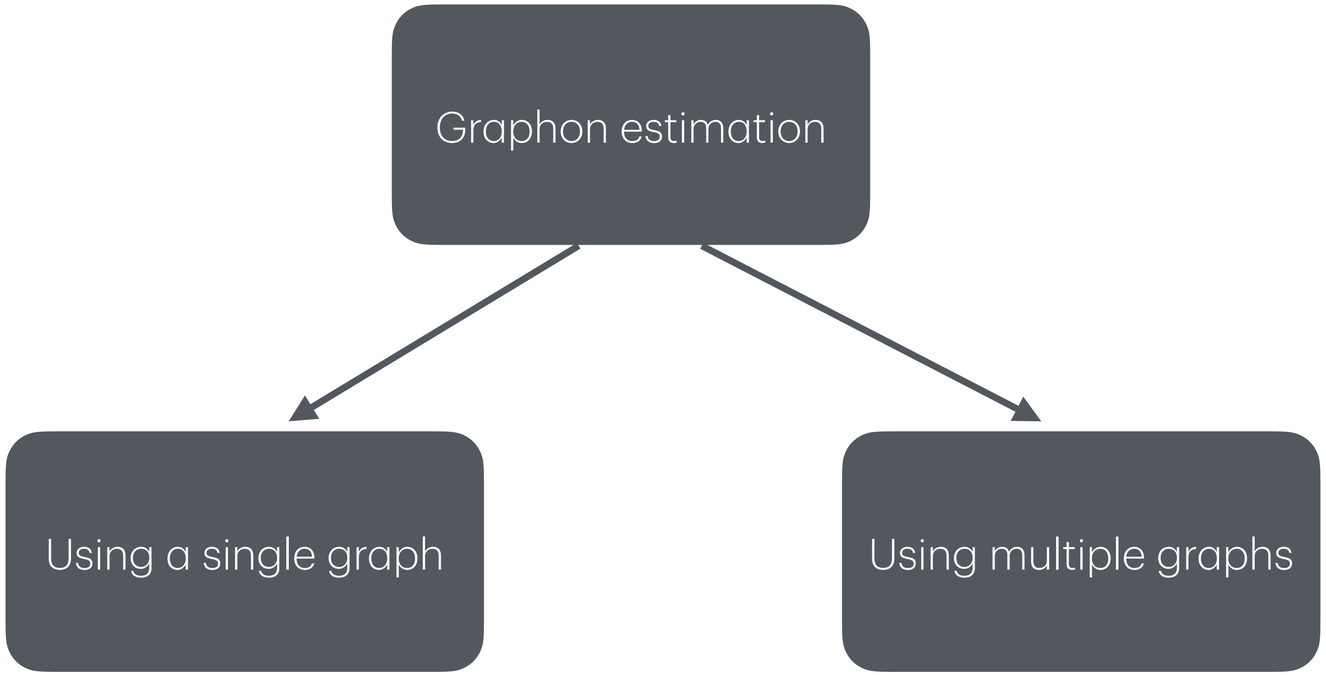
Graphon Estimation

Sevvandi Kandanaarachchi, Cheng Soon Ong
CSIRO

NeurIPS Education Track 2026

Why estimate?

- Recover the latent structure
- Given a graph, simulate growth and generate a larger graph
- Prediction purposes



- Easier estimation problem
- Only 1 sample from a graphon

- Difficult estimation problem (nodes not aligned)
- Multiple samples from graphon

Using a single graph

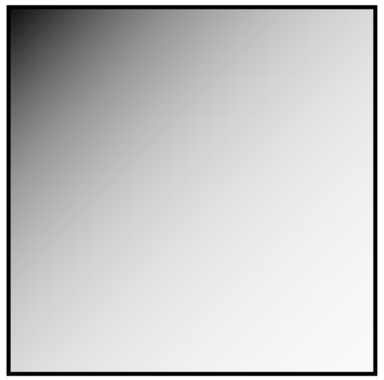


Monotonic degree condition

- Degree function $g(x) = \int_0^1 W(x, y)\, dy$

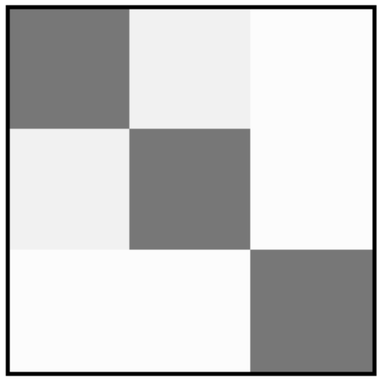
- Condition/Assumption: There exists a measure-preserving bijection ϕ such that the degree function g_ϕ associated with W^ϕ (where $W^\phi = W(\phi(x), \phi(y))$) is **strictly** increasing.

Monotonic

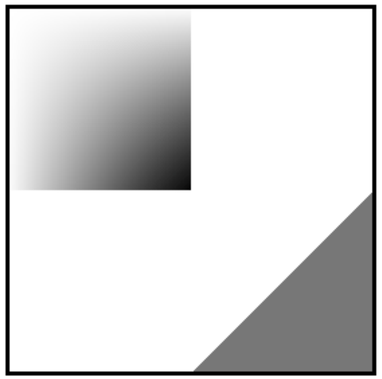


$W(x, y) = \exp(-2(x + y))$

Non-monotonic



Non-monotonic



Janson, S. (2020). A graphon counter example. *Discrete Mathematics*, *343*(6), 111836.

Monotonic degree condition - Why?

- Makes it a lot easier to estimate the graphon
- Sort the degrees => Then estimate!
- Chan, S., & Airoldi, E. (2014). A consistent histogram estimator for exchangeable graph models. In *International Conference on Machine Learning* (pp. 208-216). PMLR.

The sorting and smoothing algorithm

- Graphon estimation using a single graph
- Sort the graph by degree
- Smoothing
  - Network histogram estimation
  - Total variation minimization

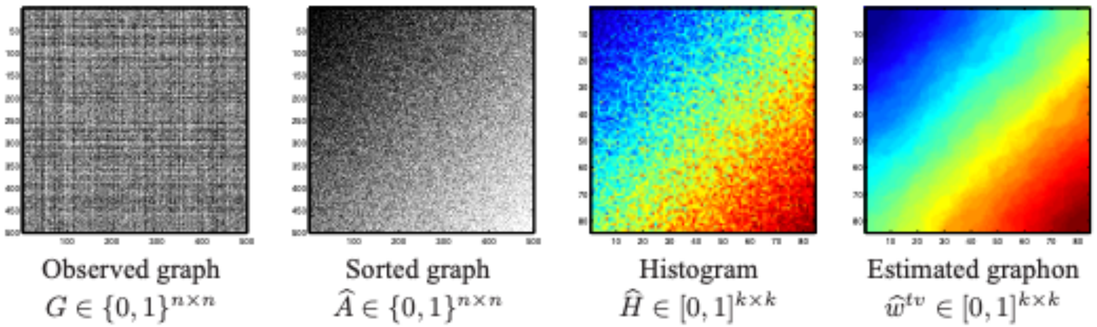


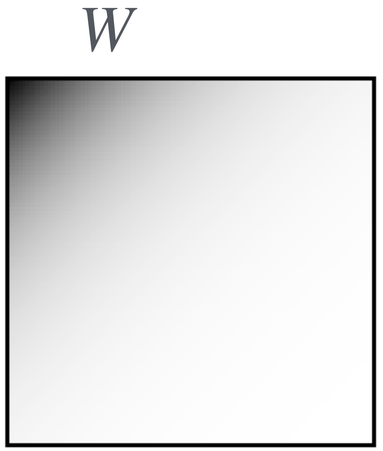
Figure 2. Illustration of the SAS algorithm. Given an observed graph G, we first sort G using the empirical degrees to get $\widehat{A}$. Then, a local histogram $\widehat{H}$ is computed and a total variation minimization is used to determine an estimate $\widehat{w}^{tv}$.

Chan, S., & Airoldi, E. (2014, January). A consistent histogram estimator for exchangeable graph models. In *International Conference on Machine Learning* (pp. 208-216). PMLR.

Using multiple graphs



Multiple graphs: node alignment between graphs



- Surprisingly, the estimation problem is harder
- $G_1 \sim W$, suppose node labels are ordered from $[0,1]$ on graphon axis
  - Degrees decrease with the node labels
- $G_2 \sim W$, suppose node labels are all random from $[0,1]$ on graphon axis
  - Degrees are all over the place
- To estimate the graphon, you need to relabel nodes so that labeling is consistent with the graphon

Popular fix: Assume graphon has monotonic degree property

Two selected methods

- Joint Graph Sorting (JGS)
  - Extends Sorting and Smoothing Algorithm by Chan and Airoldi
- Implicit Graphon Neural Representation (IGNR)
  - SIREN + GW distance

Sogan, R. B., & Rebafka, T. (2026). Low-Complexity and Consistent Graphon Estimation from Multiple Networks. In *International Conference on Artificial Intelligence and Statistics*

Xia, X., Mishne, G., & Wang, Y. (2023). Implicit graphon neural representation. In *International Conference on Artificial Intelligence and Statistics* (pp. 10619-10634). PMLR.

Joint Graph Sorting (JGS)

- Degree Monotonic assumption applies
- Take union of nodes and sort by normalized degree
- Merged adjacency matrix
- Histogram estimator
- Optional smoothing step

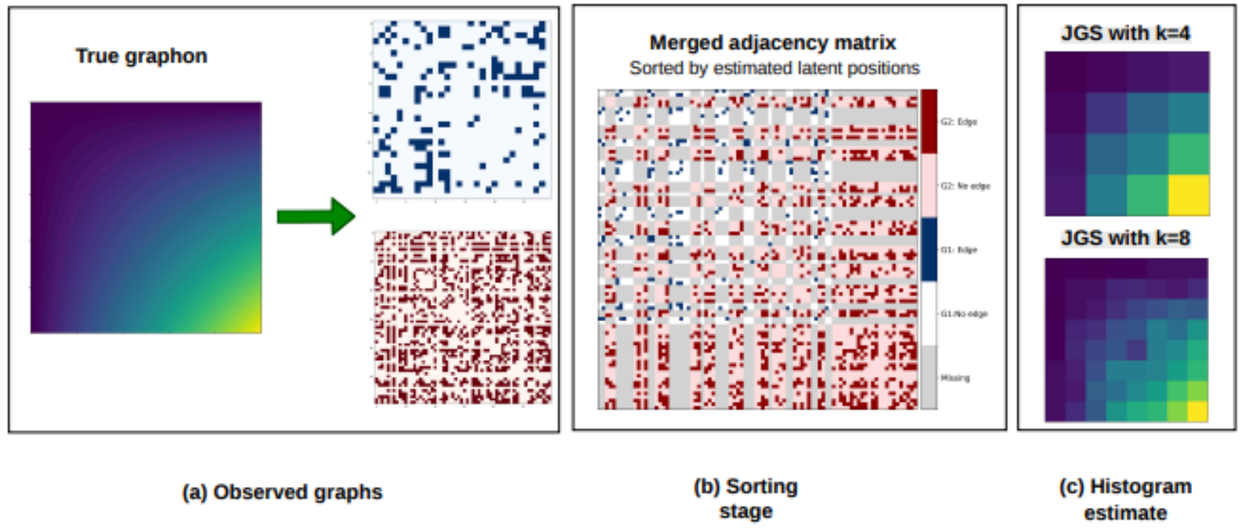
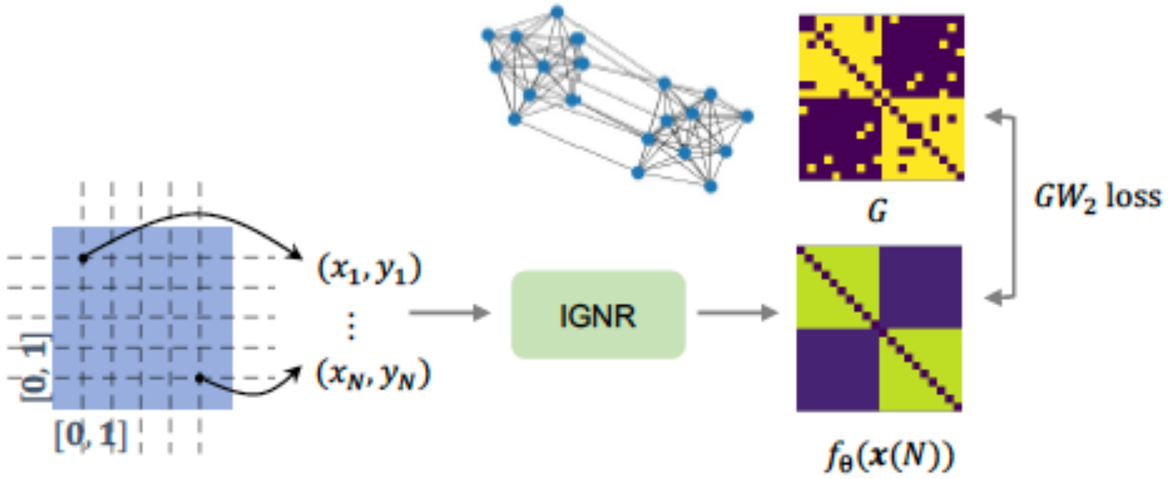


Figure 1: Illustration of the Joint Graph Sorting (JGS) estimator. (a) Networks are generated from a graphon. (b) Nodes are jointly sorted across networks using normalized degrees, yielding a merged adjacency matrix with missing entries (grey). Dark blue and white denote edges and non-edges in the first graph, respectively, while dark red and misty rose denote edges and non-edges in the second graph, respectively. (c) JGS graphon estimates with different resolutions k, where colors represent estimated edge probabilities.

Sogan, R. B., & Rebafka, T. (2026). Low-Complexity and Consistent Graphon Estimation from Multiple Networks. In *International Conference on Artificial Intelligence and Statistics*

Implicit Graphon Neural Representation

- No degree monotonicity assumption
- SIREN implementation (sine activation functions)
- $$\min_{\theta} \frac{1}{M} \sum_{i=1}^{M} GW_2(f_\theta(\mathbf{x}(N_i)), G_i)$$
- $f_\theta(\mathbf{x}(N_i))$ is a weighted adjacency matrix sampled from IGNR
  - Use optimal transport



Xia, X., Mishne, G., & Wang, Y. (2023). Implicit graphon neural representation. In *International Conference on Artificial Intelligence and Statistics* (pp. 10619-10634). PMLR.